

Power measurement and its theoretical background

This is a short description of some „well known“ values which are very often used, but in fact only very few people can remember, where they come from and what is their exact definition.

DC values

Pure signals

The most simple case of a signal is a signal which doesn't change its amplitude in any time interval.

The definition of a DC signal is quite simple:

A DC signal has always the same amplitude

In theory the „always“ is running for an infinite time. So in practice you define a time interval and say:

A DC signal has during a time interval the same amplitude

For example this time interval is defined by the time a signal is applied or by the time you need to measure the signal.

Mixed signals

There is another possible definition for a DC signal (or better the DC component of a signal). As you know, you can split every signal according to Fourier into many sinusoidal signals of different frequencies, amplitudes and phase shifts. One special case of this „sinusoidal signals“ is the DC part of a signal (which could be described as a cos function with frequency 0Hz).

According to this, the definition of the DC component is just the average value of a signal. As a formula this can be written as (in this and all other example we use the voltage. If you want to use the current, you will get to the same result):

$$U_{DC} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} u(t) * dt$$

Formula 1: General definition of the DC component

Of course this definition is also valid for pure DC signals. By this definition the user can choose any time interval to get a DC value. But in practice you have very often periodic signals. These signals are repeated after the period time T. So if T is included into Formula 1 for the time interval, the commonly known formula appears:

$$U_{DC} = \frac{1}{T} \int_{t=0}^T u(t) * dt$$

Formula 2: DC component of periodic signals**Conclusion**

- The DC component is the average value of a signal
- In practice you have to define a time interval for a DC value

Rectified values

The rectified value is usually an historical value. Today there are very few applications, in which it is needed (for example U_{rect} is proportional to the magnetic flux B_{pk} . This is important in some transformer applications).

The rectified values were created, when the first AC signals appeared. At this time there were many moving coil meters used. But they measure just the average value (DC value) which is zero for sinusoidal signals! The first simple way to solve the problem was to rectify the signal and to take the average value of the rectified signal:

$$U_{RECT} = \frac{1}{t_2 - t_1} \int_{t=t_1}^{t_2} |u(t)| * dt$$

Formula 3: General definition of the rectified component

By this definition the user can choose any time interval to get a RECT value. But in practice you have very often periodic signals. This signals are repeated after the period time T. So if T is included into "Formula 3" for the time interval, the commonly known formula appears:

$$U_{RECT} = \frac{1}{T} \int_{t=0}^T |u(t)| * dt$$

Formula 4: RECT component of periodic signals

The disadvantage of the rectified value is: For many applications you need a value which is proportional to the power which will be absorbed in a resistor (see "RMS Values"). For this you can calculate a proportional factor between the RECT value of a signal and the RMS value of a signal. This factor is called the form factor. The form factor depends very much on the wave shape of the signal! So in the scale of the moving coil instruments the form factor for sinusoidal signals was integrated. For other wave shapes you get measuring errors!

Conclusion

- The RECT component is the average value of the rectified signal
- In practice you have to define a time interval for the RECT value
- For every wave shape the RECT value has another proportional factor to get the RMS value

Form factor

The form factor is just the ratio of RMS value of a signals to the RECT value:

$$U_{ff} = \frac{U_{RMS}}{U_{RECT}}$$

Formula 5: Definition of the form factor

For sinusoidal signals it is about 1.11.

RMS values

If you have any voltage or current signal and apply it to an ohmic resistor you will have a loss power in this resistor. The problem is that every different wave shape will produce another loss power so you would have to define endlessly allowed wave shapes which will not overload the resistor. The definition of a wave shape might be quiet complex: You need a drawing or a mathematical description.

For a more practical solution it was tried to describe a wave shape via the power. This is the most general definition of the root mean square value:

The RMS value of a signal is the amplitude of a DC signal which will transfer the same energy like the applied signal into an ohmic resistor in the same time.

As a formula you could use (in this example we use the voltage. If you want to use the current, you get the same result):

$$\frac{U_{RMS}^2}{R} * (t2 - t1) = \int_{t=t1}^{t2} \frac{u(t)^2}{R} * dt$$

$$U_{RMS} = \sqrt{\frac{1}{t2 - t1} \int_{t=t1}^{t2} u(t)^2 * dt}$$

Formula 6: General definition of the RMS value

Using this definition you can choose any time interval of any signal to calculate the RMS value. If you would apply in the same time interval a DC signal with the amplitude of the RMS value the same energy would have been pushed into the resistor.

In this definition the user can choose any time interval to get a RMS value. But in practice very often you have periodic signals. These signals are repeated after the period time T. So if T is included into "Formula 6" for the time interval, the commonly known formula appears:

$$U_{RMS} = \sqrt{\frac{1}{T} \int_{t=0}^T u(t)^2 dt}$$

Formula 7: RMS definition for periodic signals

The advantage of this formula is, that you get the same value for each period. So it is sufficient to give one single value to describe a signal concerning its power consumption in an ohmic resistor. This is valid for 99% of all measuring applications. Keep in mind, that this is a simplified definition for periodic signals! Sometimes it is difficult to find the correct time interval of a signal or the question arises: What is the true time interval? This depends very much on your application.

Conclusion

- The RMS value is proportional to the power consumption in an ohmic resistor
- Usually it is defined over one period of the signal
- Other time intervals might be possible

TRMS values

In theory the RMS and the TRMS (true RMS) value are of course the same. In practice there are different measuring methods.

- Some older analogue instruments just measure the rectified value and multiply it with the form factor 1.11 to get the RMS value of the signal. This is only valid when the signal is sinusoidal. For non sinusoidal signals you need another form factor.
- Some instruments measure the AC part of a signal only. The DC component produces a power consumption in a resistor too. These instruments might display a wrong value, if a DC component exists.

To indicate that an instrument measures the RMS value independent from the wave shape of the signal, this instruments call their value the true RMS (TRMS) value.

Peak values

The definition of the peak value is simple:

the peak value is the largest amplitude of a signal

In practice there are some further definitions usual:

- positive peak value (in fact the largest value of the signal, even if it should be negative)
- negative peak value (in fact the smallest value of the signal, even if it should be positive)
- the peak value (the largest value of the amounts of the positive/negative peak values)
- the peak peak value (the difference between positive and negative peak value)

The peak value is important for measuring instruments. If it is too large for the chosen measuring range it could happen that:

- analogue instruments could come into saturation, which would cause measuring errors
- the full scale of the ADC digital instruments could be reached. Thereby the signal is cut off which cause distortions and measuring errors.

Crest factor

The crest factor is the ratio between peak value and TRMS value of a signal.

$$U_{eff} = \frac{U_{PEAK}}{U_{TRMS}}$$

Formula 8: Definition of the crest factor

For analogue instruments a maximum crest factor is defined. If the signal has a larger one, you might get measuring errors.

For digital instruments usually you find two values for a measuring range in the technical data of the user manual:

The nominal value (e.g. 250V). This can be used to select easily the correct range. The other value is the allowed peak value in this range (e.g. 400V). With these two values you can calculate your actual allowed crest factor. In this example it would be: $400V/250V=1.6$

If you apply in the same range only a 180V signal a crest factor of $400V/180V=2.2$ is allowed. So the crest factor is not that important for digital instruments as for the older analogue ones, because you can influence it.

Definitions of power

For power measurement it is important to take first a look at the energy. In a simple system you have a source and a load. Now you can transfer electrical energy from the source to the load. This energy can be absorbed or converted to another kind of energy (light, heat, rotation, ...). This is called active energy. Beside of this it can be stored and returned to the source later. This is called reactive energy.

Power is defined by energy transfer per time. So in a defined time you can transfer power to a load. Parts of this power may return to the source (reactive power), others do not (active power).

Electrical power can be divided in four groups:

Instantaneous power: This is the power at one point of time. It is calculated by multiplying the instantaneous voltage and current value at this point of time. A sequence of instantaneous power values creates the oscillation of the power. The letter for this power is “ $p(t)$ ”. So the formula is:

$$p(t) = u(t) \cdot i(t)$$

Formula 9: Definition of instantaneous power

Apparent power: This is the product of TRMS values of voltage and current. A phase shifting or a distortion of this signals are not considered. It's formula is:

$$S = U_{\text{trms}} \cdot I_{\text{trms}}$$

Formula 10: Definition of apparent power

Active power: This is the power which is transferred to the load and does not return while a defined time. So the active power is the average value of the power oscillation. It's formula is:

$$P = \frac{1}{T} \int_{t=0}^T (u(t) \cdot i(t)) dt$$

Formula 11: Definition of active power

The sign of P is “+” or “-”, depending on the direction of power flow. Active power can only be generated by same-frequent voltage and current components!

Reactive power: This is the power which is transferred to the load and does return within a defined time. The reactive power is calculated as:

$$Q = \sqrt{S^2 - P^2}$$

Formula 12: Definition of reactive power

The reactive power can have two causes:

- a phase shift between voltage and current components with the same frequency causes the “phase shift reactive power” called Q_{shift} and calculated as follows:

$$Q_{\text{shift}} = \sum_{i=1}^n U_i \cdot I_i \cdot \sin \varphi_i$$

Formula 13: Definition of phase shift reactive power

Phase shift reactive power can only be generated by same-frequent voltage and current components!

- the combination of voltage and current components with different frequencies causes the “distortion reactive power” called D or Q_{dist} .

The geometrical addition of this two kinds of reactive power is the total reactive power:

$$Q = \sqrt{Q_{\text{shift}}^2 + Q_{\text{dist}}^2} = \sqrt{Q_{\text{shift}}^2 + D^2}$$

Formula 14: Definition of total reactive power

which is also calculated with the above formula.

For a better understanding the following figure is maybe useful:

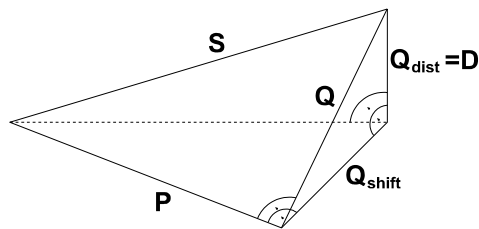


Figure 1: Graphical relation of the different powers

The distortion reactive power D of a system can be calculated with the formula:

$$D = \sqrt{S^2 - P^2 - Q_{\text{shift}}^2}$$

Formula 15: Definition of distortion reactive power

Two simple rules:

1st Rule

Same frequent voltage and current components can produce active power and phase shift reactive power only.

The power waveform can be described with following formula:

$$p(t) = \hat{i} * \sin(\omega t + \varphi_i) * \hat{u} * \sin(\omega t + \varphi_u) = \hat{i} * \hat{u} * \left(\frac{1}{2} \cos(\varphi_i - \varphi_u) - \frac{1}{2} \cos(2\omega t + \varphi_i + \varphi_u) \right)$$

Formula 16: power waveform of same frequent signals

As told above (see Formula 11), the active power is the average value of the power waveform. While taking a look to the above formula you see, that only the term

$$\hat{i} * \hat{u} * \frac{1}{2} \cos(\varphi_i - \varphi_u)$$

has an average value which can be different from zero! This depends on the phase angles of the voltage and current signal. The other term

$$\hat{i} * \hat{u} * \frac{1}{2} \cos(2\omega t + \varphi_i + \varphi_u)$$

can never produce an active power, because it's average value over one period is zero!

The first term can be modified to give you a common known formula:

$$\begin{aligned} P &= \hat{i} * \hat{u} * \frac{1}{2} \cos(\varphi_i - \varphi_u) \\ &= \frac{\hat{i}}{\sqrt{2}} * \frac{\hat{u}}{\sqrt{2}} * \cos(\varphi_i - \varphi_u) \\ &= I * U * \cos(\varphi) \end{aligned}$$

Formula 17: Active power of same frequent signals

Pay attention that this formula is correct for voltage and current components with same frequency only!

2nd Rule

Components with different frequencies will produce distortion reactive power

Again the power waveform can be described with following formula:

$$\begin{aligned} p(t) &= \hat{i} * \sin(\omega_1 t + \varphi_i) * \hat{u} * \sin(\omega_2 t + \varphi_u) \\ &= \hat{i} * \hat{u} * \left(\frac{1}{2} \cos((\omega_1 - \omega_2) * t + \varphi_i - \varphi_u) - \frac{1}{2} \cos((\omega_1 + \omega_2) * t + \varphi_i + \varphi_u) \right) \end{aligned}$$

Formula 18: power waveform of different frequent signals

In this case you see, that the first term as well as the second term will have an average value over one period of zero! So both terms can never produce any active power, just distortion reactive power. The maximum period time of this signal can be calculated according to following formula:

$$T = \frac{1}{\text{gcd}(f_1, f_2)}^1$$

Formula 19: power waveform period time of different frequent signals

In all the above definitions we talked about a “defined time”. With periodical signals this time is an integer number of complete periods (because after a complete period the signal is periodical and the values of different measurements are identical. If you would not measure for a complete period the resulting values are not the same in the next measurement). If you use more than one period for measuring you can get an averaging over this measuring time. For this reason you can set up a basic “**cycle time**” in the ZES ZIMMER® instruments. This cycle time determinates the average over some periods.

Non periodical signals like starting characteristics are not that easy to measure because it is very difficult to determine the “defined time”. For example if you start up an elevator, you transfer energy to the motor, which is the load. Now the motor runs for some minutes and is stopped then. While stopping the motor recovers some energy to the source. This energy is reactive energy. If you measure with cycles of 1s you measure the transfer to the motor while start up as well as the transfer from the motor while stopping as active energy because your “defined time” is not long enough to take the shut down into calculation. So while measuring non periodic signals there is a possible error.

Of course you can measure many short sequences of TRMS and active power values. This values are correct in the “defined time” which is the measuring time. Only the user can decide if some of the power will return later.

Power factor

The power factor is the ratio of active power to apparent power:

$$\lambda = \frac{|P|}{S}$$

Formula 20: power factor

A second value which is very often confound with the power factor is the $\cos(\varphi)$. It has the same definition, but it is only valid, if **voltage and current are both** sinusoidal! Therefore the power factor is a more general definition.

Sometimes the cosine of the angle between the fundamental voltage and the fundamental current is sometimes called Displacement Power Factor (DPF).

The above ratio between P and S is sometimes called True Power Factor (TPF).

Multi phase systems

Total (sum) values

¹ gcd is the greatest common divisor. This function is only defined for integers. If otherwise, then the fraction has to be multiplied with a common factor.

E.g. for $f_1 = 50$ Hz and $f_2 = 50.2$ Hz the result is $T = 10/\text{gcd}(500,502) = 5$ s

All the above values for a single phase are well defined. For example in a three phase system the situation becomes more complex:

The total (or sum) active power is the simple sum of all phase active powers. No problem so far. With the apparent power we face the first problem. This is not any vector! Only the apparent power component which is caused by the phase shift (see the dashed line in Figure 1: Graphical relation of the different powers) could be added as two dimensional vectors. But there is also a component caused by distortion reactive power which is not a vector. So it's wrong to just add the apparent power of different phases. It's also wrong to add them as a two dimensional vector. The same problem happens with the reactive power. There is an important difference between Q_{shift} and Q_{dist} !

A further problem and it's solution could be explained very easily by the power factor:

In general it can be said, that the power factor is a value which expresses the usage of a power distribution system. If you have any kind of reactive power (independent if phase shift reactive power or distortion reactive power) you will get a value smaller than 1.

Also in a three phase system you will get a total power factor smaller than 1 if you have just ohmic loads which are not equal. In this case you have not used the system in an optimum way, because the load is not symmetrical and one phase has to drive more current than the other ones. The power factor of each phase is of course 1!

Example:

Phase 1: 230V, 1A, ohmic load. $P=230\text{W}$, $S=230\text{VA}$, $\lambda=1$

Phase 2: 230V, 2A, ohmic load. $P=460\text{W}$, $S=460\text{VA}$, $\lambda=1$

Phase 3: 230V, 3A, ohmic load. $P=690\text{W}$, $S=690\text{VA}$, $\lambda=1$

Collective sum voltage according DIN40110-2:

$$U_{\text{trms}} = \sqrt{U1_{\text{trms}}^2 + U2_{\text{trms}}^2 + U3_{\text{trms}}^2}$$

Formula 21: Collective sum voltage

$U_{\text{trms}}=398.37\text{V}$

Collective sum current according DIN40110-2:

$$I_{\text{trms}} = \sqrt{I1_{\text{trms}}^2 + I2_{\text{trms}}^2 + I3_{\text{trms}}^2}$$

Formula 22: Collective sum current

$I_{\text{trms}} = 3.742\text{A}$

Total system: 398.37V, 3.742A, ohmic loads, $P=1380\text{W}$, $S=1490.7\text{VA}$, $\lambda=0.926!$

At this system the total power factor is smaller than 1 because the phases drive different loads.

Modern ZES ZIMMER® measuring instruments work according to DIN40110-2. It's the only standard which describes this kind of systems in a sufficient way.

Sometimes values like “sum voltage” or “sum current” are requested, in the meaning like “U1+U2+U3” resp. “I1+I2+I3”. These values have no physical background (in opposite to the collective sum current according DIN40110-2). A customer can calculate these values with the implemented formula editor.

Special wirings, star to delta conversion

While measuring phase to neutral conductor voltage of a phase and with the same power measuring channel the current in this phase, you get the power consumption of this phase. If you do this with several phases you can get the total values of this system like written above. However there are some applications, where you have no access to the neutral conductor. For these applications you can use special wirings and features of ZES ZIMMER® power meters.

Aron circuit

This circuit can be used in three phase systems with three wires. If you have a 4th wire, it can't be used because currents could flow in this wire, which are not taken into account. Please note, that in high frequent applications ground could be the 4th “wire”!

The main advantage of this circuit is, that you just need 2 power meters to measure the **total active power** of the system.

You measure the current in phase 1 and 3 as well as the voltage of phase 1 resp. 3 against phase 2.

With this you apply a linked voltage and a phase current to each measuring channel. Just with an ohmic load the channel sees a phase shift of 30°. With non-ohmic loads the displayed active power of each channel can be larger, smaller or even negative.

The active power of each phase can't be displayed with this, also the reactive and apparent power as well as the power factor. Only the **total active power** is correct (and of course the voltages and currents). All other total values are invalid.

Derivation

Following you find the (very much simplified) derivation for the measuring method:

$$P = U_1 * I_1 + U_2 * I_2 + U_3 * I_3$$

$$U_{12} = U_1 - U_2 \text{ gives } U_1 = U_{12} + U_2, \text{ likewise}$$

$$U_{32} = U_3 - U_2 \text{ gives you } U_3 = U_{32} + U_2.$$

If you insert this in above formula you get:

$$P = U_{12} * I_1 + U_2 * I_1 + U_2 * I_2 + U_{32} * I_3 + U_2 * I_3$$

$$P = U_{12} * I_1 + U_{32} * I_3 + U_2 * (I_1 + I_2 + I_3)$$

In a three wire system the vector sum of the currents must be zero ($I_1+I_2+I_3=0$), which gives you:

$$P = U_{12} * I_1 + U_{32} * I_3$$

That's exactly the measuring condition which was described above. Of course this derivation can be done in a universal way.

Special features in LMG450

As a special feature the LMG450 calculates the third (not measured) current (in phase 2) and the third, not measured voltage U_{31} . This values are correct, the power values and other derived values not (see above).

With the option star to delta conversion, the "wrong" values of aron circuit can be recalculated to really existing star or delta circuit values. As a result you get active, reactive and apparent power as well as power factor for each phase as well as for the total system!

Measuring delta voltages and star currents

A very typical situation when measuring motors is that you have only L1, L2 and L3 to the motor, but no neutral conductor. So without additional tools you can measure the voltage U_{12} , U_{23} and U_{31} together with the currents I_1 , I_2 and I_3 only.

Similar like in the aron circuit you apply a linked voltage and a phase current to each measuring channel. Just with an ohmic load the channel sees a phase shift of 30° . With not ohmic loads the displayed active power of each channel can be larger, smaller or even negative.

Worse then in the aron circuit, here only the voltages and currents in each channel are correct, but all other values, also all total values are wrong.

The LMG instruments offer a so called star to delta conversion. If you connect the instrument like mentioned above, you can choose two special wirings:

- a) 3+1, $U_{\Delta I^*} \rightarrow U_{\Delta I \Delta}$ The values are recalculated in a way to get the values in a delta circuit.
- b) 3+1, $U_{\Delta I^*} \rightarrow U^* I^*$ The values are recalculated in a way to get the values in a star circuit.

b) is the more common one but both have the same positive result: the LMG calculates now values which belong together: U_1 , I_1 , U_2 , I_2 and U_3 , I_3 respective U_{12} , I_{12} , U_{23} , I_{23} and U_{31} , I_{31} . Thereby it is possible that **all values** from **all channels** including the **total values** are correct.

Times

In the above paragraphs it was told about a lot of times: period time, cycle time, measuring time,

Here is the explanation of different times which run in an instrument. Combined with this also the synchronisation and some other important things are explained.

Sampling time

The fastest time is the time between two sample values. This time depends on the number of conversions per second. For example at the LMG95 we have about 100000 conversions per second (for each voltage and current channel) and so every 10 μ s we see a new sample value.

Synchronisation

You have to choose which signal you want to synchronise. Why do you have to synchronise at all?

As described for example in section "RMS values" were the values are defined for a defined observation time. For periodic signals this is the period time of the signal (or multiple of period times). So the instrument has to measure for a multiple number of periods of the signal. In the LMG series usually the positive zero crossings are used to detect the end of a signal period.

Cycle time

The cycle time defines after which time you want to get measuring results. As written above the instruments measures for a multiple number of periods. How you can handle this?

Let's assume you have a 50Hz signal (20ms period time) and a 50ms cycle time. Exactly when the cycle time starts you have the start of a 20ms period too. The instrument starts measuring. After 50ms the cycle ends. The instrument has measured from t=0 to t=40ms exactly 2 periods of the signal. This values are calculated to all displayed values like Utrms, Idc, P,

The exact measuring time in this cycle was just 40ms!

For the second cycle all sample values from t=40ms are taken (our instruments measures without any gaps!!!). At t=100ms the second cycle ends. At this time the period of the signal ends. Now the instrument takes the sample values from t=40ms to t=100ms to calculate the values. The exact measuring time in this cycle was 60ms! Here 3 periods were measured. This cycle has had another time interval. For periodic signals it is not important because each period is exactly the same! Fluctuating signals can have influences on the results.

You see: The cycle time defines how often you want to get values (in average). The exact time is defined by the synchronisation signal!

Another important requirement is, that the cycle time has to be bigger than the period time! It is not possible to measure a 1Hz signal in 500ms!

Integration time

To perform an energy measuring (which usually takes a longer time: some minutes to several weeks) you have to define an energy measuring time too. This time has always to be an integer of the cycle time, because the instrument can take complete cycles into energy calculation, only.

Observation time for harmonics

When calculating harmonics you use the sample values over an integer number of periods. The inverse value of this window width is the frequency step size in which the harmonics are calculated.

Example: While measuring a 50Hz signal for 20ms (1 cycle), you got harmonics in 50Hz steps. While measuring 16 cycles (320ms) you get harmonics in 3.125Hz steps.

By choosing a different observation time you can define how many inter harmonics you will get.

The observation period is derived from the synchronisation signal, too.

Undersampling, Shannon theorem, anti aliasing filter

Concerning the bandwidth and the sampling frequency there is a very frequently asked question:

How can you measure a 500kHz signal while having just 100kHz sampling rate?

Many people assume that the sampling theorem (also called Nyquist or Shannon theorem) has to be met to calculate values. This is not true except for few purposes, which are relevant for our kinds of measuring:

- you want to analyse the signal in the time domain (like in the scope function)
- you want to filter digitally (e.g. IIR or FIR filter)
- you want to make a FFT or DFT

For our purposes of measuring DC, TRMS or power values it has not to be met! The reason for that is simple:

All these values are in principle statistical values (average value, squared average value, ...). So it doesn't matter in which order you add the values to get the sum (or square sum).

A simple example:

You have a sine wave of 50Hz, amplitude 1 and a sampling frequency of 200Hz. You sample the first period of the sine wave with following results (phase angle/amplitude): $0^\circ/0$, $90^\circ/1$, $180^\circ/0$ and $270^\circ/-1$. So the sum is zero.

Now let's take the same signal, but a sampling frequency of 40Hz and we take several periods. Here are the results (period/phase angle/amplitude): $1/0^\circ/0$, $2/90^\circ/1$, $3/180^\circ/0$, $4/270^\circ/-1$. You see you need 4 periods, but you get exactly the same sample values.

Now let's assume a sampling frequency of 28.6Hz ($=1/0.035s$) and again we take several periods. Here are the results (period/phase angle/amplitude): $1/0^\circ/0$, $2/270^\circ/-1$, $4/180^\circ/0$, $6/90^\circ/1$. You see that the order of the sampling values has changed, but the sum is the same again!

You will get the same statistical values of a wave shape, independent of the sampling frequency. Important is, that each point of the wave shape was sampled, which might take several signal periods to get this. This working principle is called undersampling.

Coming back to the question: the shortest cycle time is 50ms. So a 500kHz signal ($2\mu s$ period time) is sampled for at least 25000 periods! With longer cycle times, you also have more signal periods included.

As mentioned above you have to meet the Shannon theorem, if you want to perform a Fourier transformation for example. In principle the bandwidth of your signal has to be smaller than half of the sampling frequency. So with 100kHz sampling frequency the signal bandwidth should not exceed 50kHz. If it is larger you might get aliasing (which means you see signals at wrong frequencies with wrong amplitudes). In practice the standard IEC61000-4-7 defines a rejection of -50dB as sufficient to get no aliasing.

In principle there are two possible ways to meet this requirement:

- The uncertain way is when a signal has a (theoretically) small bandwidth. Typically AM radio stations transmit with low bandwidth. This may cause aliasing due to coupling.
- The safe way is to use an anti-aliasing-filter inside the instrument. This also rejects the components which are coupled externally into the signals. The instrument itself is shielded, so that no problems occur.

This anti aliasing filter (in the instrument we call it a “HF-rejection” filter) is automatically used inside the instrument, if you select a harmonic mode. Of course this filter has not an infinite steepnes (0db<50kHz, -50dB >50kHz). So you can measure harmonics up to about 10kHz. In the range from 10kHz to 50kHz the filter derates down to -50dB.

Calculation of apparent power with LMG600 and following generations

The calculation of EQ and ES in the LMG600 is done slightly different compared to older device generations. For reference see the equations in the manual.

With the old calculation method there was a possibility that the values of ES and EQ would have been dependant of the cycle time. When several cycles are coupled into one energy-measurement-interval the complete active energy EP stays, independant from the cycletime, constant, but the apparent energy ES is dependant from the chosen cycle time. To get a better understanding of this problem you should take a look at a simple example to explain how the devices measure energy: In the course of the energy-measurement-interval the voltage drops exponentially while the active power stays constant, therefore the current rises. We assume a measurement cycle of one second. The values are SI standard units (energy in Ws).

1 second cycle time					
Time	0-1	1-2	2-3	3-4	LMG500
P	1	1	1	1	EP = 4
Ueff	4	2	1	0.5	
Ieff	0.5	1	2	4	
S	2	2	2	2	ES = 8
Q	1.732	1.732	1.732	1.732	EQ = 6.928

2 seconds cycle time			
Time	1-3	3-5	LMG500
P	1	1	EP = 4
Ueff	3.162	0.7905	
Ieff	0.7905	3.162	
S	2.5	2.5	ES = 10

4 seconds cycle time		
Time	1-5	LMG500
P	1	EP = 4
Ueff	2.305	
Ieff	2.305	
S	5.312	ES = 21.25
Q	5.218	EQ = 20.87

Looking at this example one can see that the power values P, Q and S are stable over time, i.e., they don't change from cycle to cycle. Nevertheless the values for S and in correspondence also Q are dependant from the chosen cycle time. This shows clearly that summing to calculate ES and EQ is dependant of the chosen cycle time.

Since the LMG600 has a much more capable signal processing hardware than its predecessors the dependance of the cycle time for the values of EP and EQ could be solved. We decided to implement an own measurement cycle for energy measurement, utilizing the modern, parallel data processing: The measurement cycle for the energy has the length of the energy-measurement-intervall. That allows the values for ES and EQ to be not dependant of the cycle time anymore, instead it is only dependant of the intervall. A nice side effect is, that now the correspondence $EP^2 + EQ^2 = ES^2$ is valid.

In practical conditions the above signals are not very common. Usually the voltage is more or less constant. When the current is also constant there aren't any differences between the two measurement methods. But when the current is changing greatly over time those differences start to become visible. One can imagine it like that: A pure resistive load is switched rarely between 100W and 200W. The power is measured. In measurement cycles where no switching occurs the apparent power is the same as the active power, since current and voltage are proportional. The power factor is 1. In a measurement cycle where the power is switched between 100W and 200W the current is not proportional to the voltage, since it isn't sinusoidal anymore because of the sudden change of the power. Distortion reactive power is created, P is now smaller than S. The power factor is smaller than 1.

Since the measurement device has only a temporal restricted view of the signal it would often show a power factor of 1 ($P=S$) and rarely smaller 1 ($P<S$). When using short cycle times (small in comparison to the switching frequency) in many cycles a small reactive power is displayed, as compared to a wider view onto the signal with a longer cycle time, as one can see in the above example. The optimum is reached when the cycle time is the same as the energy-measurement-intervall, which is the way this problem is solved in the LMG600.

You can also calculate this example with real numbers: For 100W you need 0.435A at 230V, for 200W 0.87A. If the switching point is exactly in the middle of a measurement cycle the resulting measured power is exactly the linear average value (150W), but since the effective values has a cubic character the average value is also a cubic average with 0.688A. This reduces the power factor for this measurement to 0.95.

Reactive power of a non - linear load supplied with a sinusoidal source

Two methods of measuring reactive power are shown, one using the original distorted current signal, the other a filtered current signal. In this example the device under test is a LMG95 measuring its own consumption.

Measurement with a distorted current signal

We used the following input signals:

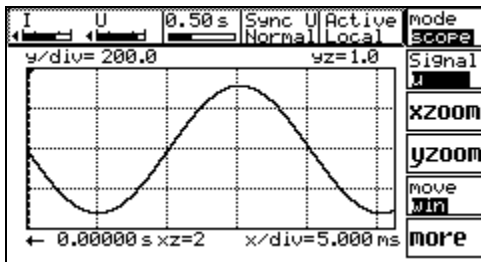


Figure 2: Voltage input signal

The voltage signal is taken from a ZES ZIMMER® 5000i power source. This voltage is sinusoidal with a frequency of 50Hz.

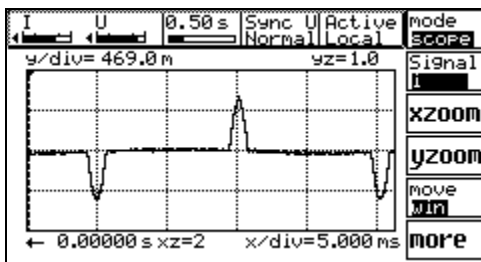


Figure 3: Unfiltered current input signal

The current signal is unfiltered and distorted, but contains only harmonic components (50Hz, 100Hz, ...).

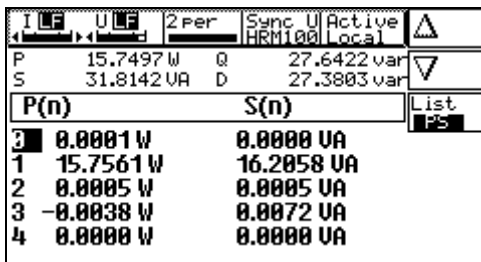


Figure 4: Power values in HARM100 mode

These are the values measured in “HARM100” mode with the above shown input signals. As you can see the apparent power “S” is very high, caused by the distortion reactive power “D” and not by the phase shift. You can calculate the phase shift reactive power Q_{shift} , which is only:

$$Q_{shift} = \sqrt{Q^2 - D^2} = 3,7961 \text{ var .}$$

Measurement with a filtered current signal

Another way to get “D” is shown in following experiment. We used the same set-up with a digital 30Hz filter inside the LMG95. The voltage input signal is taken also from the ZES ZIMMER® 5000i power source and so the same like in the first set-up.

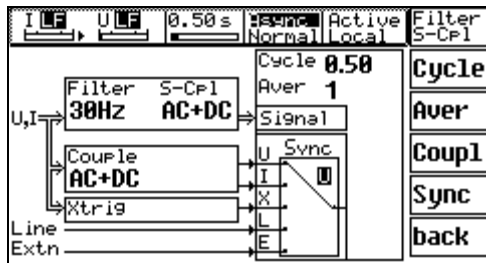


Figure 5: Filter settings

To filter the distorted current input signal we selected the above shown filter settings.

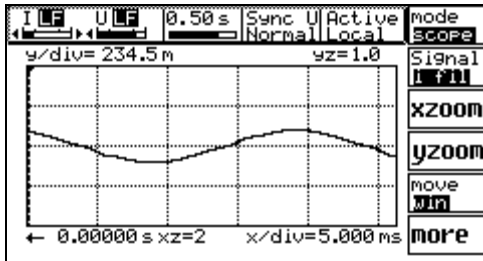


Figure 6: Filtered current input signal

Now the current wave form is now nearly sinusoidal, because all harmonics with order ≥ 2 have been rejected.

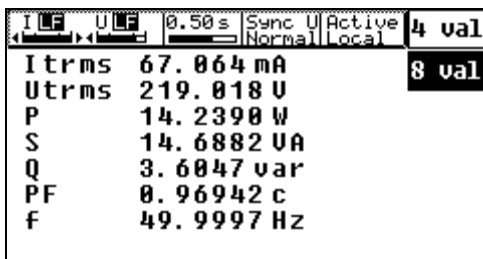


Figure 7: Measured values

Due to the filter the fundamental components are a little bit smaller. As you can see the active power “P” is nearly as large as the total absorbed apparent power “S”. Now the only measured reactive power “Q” is caused by the phase shift between voltage and current. This is nearly identical with the value, $Q_{\text{shift}} = 3,7961\text{var}$, calculated in section “Measurement with a distorted current signal”.

The distortion of one input signal may cause a very large apparent power. To calculate the distortion reactive power “D” you need two different values of reactive power “Q”. The first needs to be measured with the distorted input signals, what means the total reactive power with the two parts: Q_{dist} (D) and Q_{shift} . The second measurement needs to be performed with filtered input signals. Now the distortion reactive power “ Q_{dist} ” or rather “D” becomes zero (no harmonics), what means Q is equal to Q_{shift} . With the following formula you get the correct value of “D”:

$$D = \sqrt{Q^2 - Q_{\text{shift}}^2} \cdot \quad Q = \text{measured without filter}$$

$$Q_{\text{shift}} = \text{measured with filter}$$

Note: This works while only Q_{shift} remains after filtering. An example in which this does not work follows below.

With the LMG95 you can measure the correct value for “D” directly with the “HARM100” option (refer Figure 4: Power values in HARM100 mode).

Measuring of a burst firing control signal of a heat gun

The following breadboard will describe the difficulties of measuring the correct values of the different kinds of power. You will find a lot of the above theoretical explanations here in a practical application.

Experiment

The breadboard set-up is shown in the picture below:

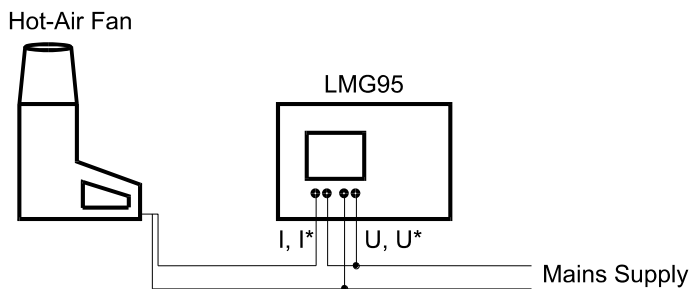


Figure 8: Breadboard set-up

Time signal

The following figure shows the time signal of the absorbed current. The peaks show the current which is needed to heat the heating coil. The signal before and after heating is the one way rectification current of the fan motor. The signal can be interpreted as an amplitude-modulated 1.5Hz rectangular signal with a 50Hz carrier frequency with neglect of the one way rectified current.

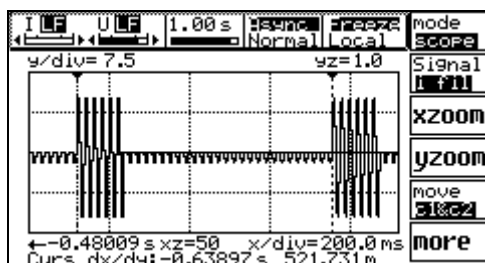


Figure 9: Measured time signal

Optimal equipment for measuring the different kinds of power is the ZES ZIMMER® LMG series with option "HARM100". The cycle time is the largest issue you can face while using ordinary instruments. The rectangular signal above has a period of 639ms; measurements with a cycle time of 1s, which means 1 to 2 beats, will get very fluctuating measurement values of the power. The problem was the trigger on U or I frequency. Because of the amplitude modulated signal the baseband signal (1.5Hz) is not existing hence triggering with common used filters is impossible.

The LMG series provides a special trigger setup to allow measuring AM-demodulation.

Measure menu set-up

The following adjustments must be set up in measure menu: Important: Xtrig!

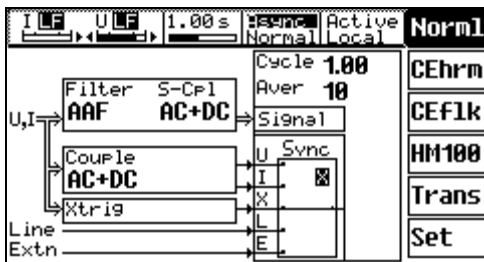


Figure 10: Measure menu set-up

XTrig menu set-up

To measure the correct values of the different powers the amplitude modulated signal must be squared and filtered with a 30Hz filter (the 100Hz squared carrier frequency will be eliminated). It is important to set the trigger level to $10A^2 \hat{=} (3,2A)$, because of the DC component after the squaring, there are no zero crossings!

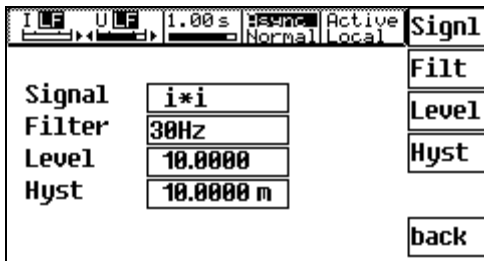


Figure 11: Filter settings in the measure menu for a squaring AM demodulator

The values in the default menu

The values shown in the default menu are correctly measured TRMS values of the burst firing control signal. The basic frequency of the signal is 1.5Hz. All values are stable! This is an indication for a correct synchronisation (like a stable signal on a scope!).

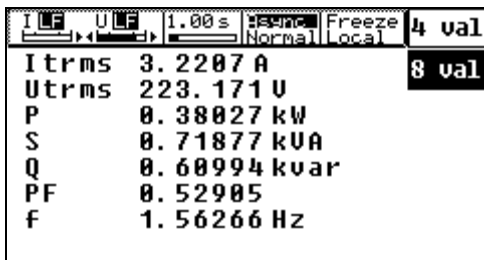


Figure 12: Values in the default menu

Harmonic analysis

For getting a better understanding of the measured values the harmonic analysis is very helpful. The following figures show the participation of the different current harmonics, which have an influence on the TRMS of the measurement, triggered on 1.5Hz fundamental.

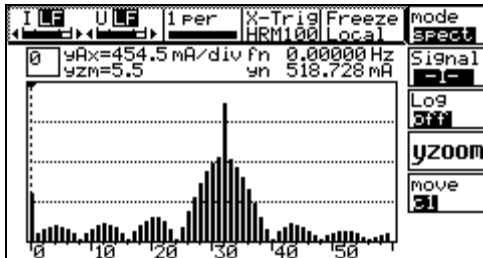


Figure 13: DC-component (cursor position), caused by the fan motor driven with halfwave rectifier

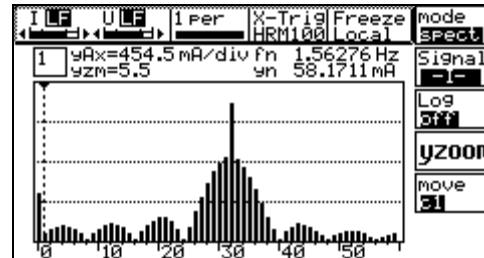


Figure 14: 1.5Hz component

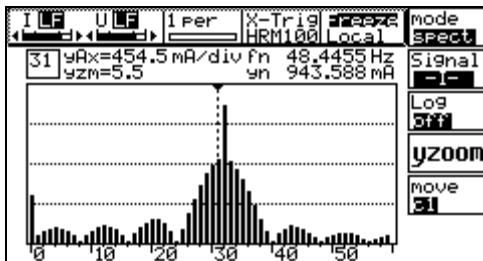


Figure 15: First frequency of the lower side band, that is the 31st harmonic of 1.5Hz fundamental

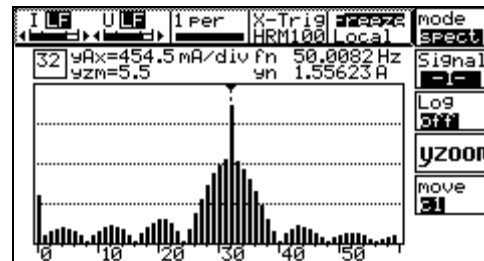


Figure 16: 32nd harmonic is the 50Hz carrier frequency

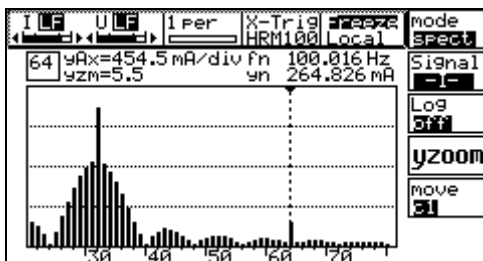


Figure 17: 64th harmonic (100Hz) means the 2nd harmonic of the fan motor

The harmonic analysis shows that the signal will have a very high content of distortion reactive power "D", because the voltage is about sinusoidal and the current has many spectral components between 0 ... 50Hz and also between 50 ... 100Hz.

Difference to the "normal" measurement: Line triggered

The following figures show the harmonic analysis of the signal, triggered with line frequency. The first figure was shot while the heating coil was not in operation. The DC component is caused by the fan motor.

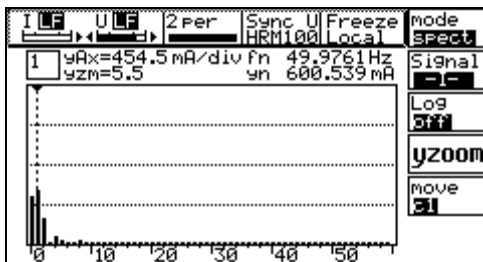


Figure 18: Measured harmonics triggered with line frequency

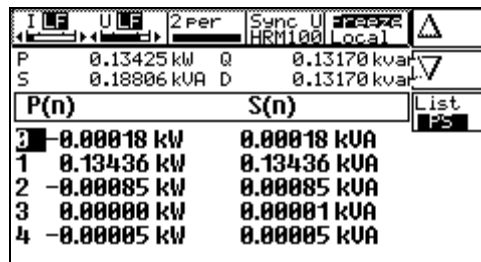


Figure 19: Active and apparent power while the heating coil is not in operation

In the second figure the heating coil started to heat, the current increases up to 8A.

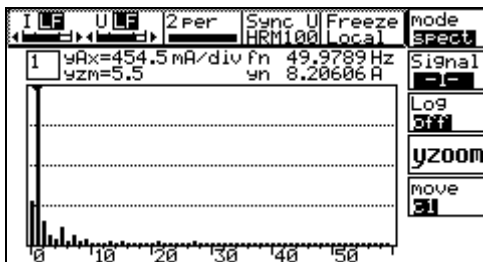


Figure 20: Heating coil in operation

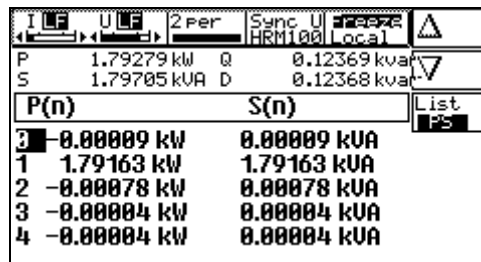


Figure 21: Active and apparent power while heating

The distortion reactive power is quiet constant, independent of the heating coil is in operation or not. In this cases “D” is caused by the one way rectification of the fan motor current. “P” and also “S” change very much depending on the signal being analysed (heating on/off). All measurements were done for 2 periods of line frequency.

Power analysis

With X-synchronisation to the frequency of the 1.5Hz modulation signal the following power values were measured:

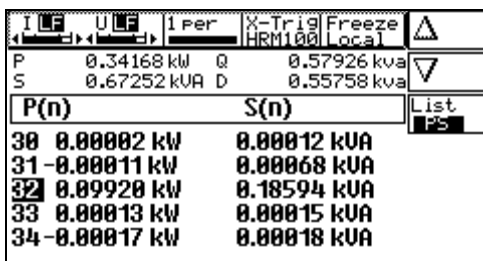


Figure 22: Active power and apparent power at line frequency (32nd harmonic)

Here the 50Hz power component is at the 32nd harmonic. The distortion reactive power is larger, because both sidebands are taken into the calculation. The values "P" and "S" are stable.

Conclusion

The integration time in which a measurement has been made is a very important parameter to get correct values. Also the interpretation of the measured signal is very important. As you can see, you can measure 3 different active powers:

1.79 kW	during the heating
0.13 kW	during the fan motor running only
0.34 kW	over one complete control cycle

All measurements are correct for their special time. Which power is the valid one for you depends on the time period you are interested in.

Usually the best synchronisation is on the signal with the lowest frequency.

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